Introduction

What began as an effort to reproduce the observations made by Christoph Amelung and Joseph Rothberg when they heated BCAMs became a long series of experiments and calculations that eventually lead us to conclude that the unusual behavior of the BCAM when it was heated did not, in fact, have anything directly to do with the heating. In the course of our investigations, we discovered a CCD whose dark current increased dramatically with temperature, we noticed bright reflections off metal parts, and we identified a particular form of defocus as the source of the original observation. In this report, we describe our experiments, and include all the calculations and analysis we think might be relevant to future investigations, even if they turned out to be irrelevant to our current investigations.

Apparatus

Our apparatus is similar to that of Christoph Amelung at CERN, who was the first to perform this experiment. We have two BCAM cameras facing one another. One camera is mounted horizontally on a kinematic mount, the other is mounted sideways on a kinematic mount and a micrometer stage so that it can be moved perpendicular to its optical axis. Both BCAM mounts are based upon sliders that can move towards and away from one another along an aluminum rail. When the two sliders touch, the BCAM lenses touch. When the two sliders are 70 cm apart, each BCAM sees both of the laser diode light sources on the other BCAM. The BCAM axes are almost coincident. During our experiments, when the two BCAMs are at the desired distance from one another, we fix the sliders in place with a fastening screw.

We have four camera-laser pairs in this arrangement and we name them as follows. The horizontal BCAM is the ‘left’ BCAM, and the sideways BCAM is the ‘right’ BCAM, for left and right as we viewed them. The image taken by left
BCAM of the top laser on the right BCAM is BCAM_LT. We also have BCAM_LB. The image taken by the right BCAM of the left laser on the left BCAM is BCAM_RL. We also have BCAM_RR.

When the BCAMs are only 70 cm apart, the images of their lasers are separated by 1.7 mm on the CCD. When they are 10 m apart, the images are separated by only 120 µm. In both cameras, the images are separated vertically when we display the images on our computer screen.

To heat the BCAMs, we used a 3-W bulb and a 150-W bulb. Our initial experiments we performed with V2A BCAMs, which dissipated 1 W internally during our data taking. Later experiments used V2B BCAMs, which dissipated only 40 mW because we could send them to sleep between measurements. Whenever we heated a BCAM, we made sure that it reached thermal equilibrium before we began heating. In the case of the V2A BCAMs, we made sure they had reached equilibrium under their own self-heating.

Initially we used an analysis threshold of 30 counts, or 5 counts above our zero-pedestal, but we found that the measured position of each image was occasionally spiking by several micrometers. We raised the threshold, and the spikes vanished. We concluded that asynchronous variation in the fluorescent background illumination and the reflected intensity of our heating lamps was compromising the effectiveness of our background-subtraction, and occasional pixels in the main body of the image would disturb our calculation of the image centroid.

3-W at 1 cm, BCAMs Separated by 70 cm

With the 3-W bulb 1 cm from the top of the left BCAM, and the BCAMs separated by 70 cm, we observed no change in the image positions in either BCAM. The standard deviation of spot position was 0.2 micrometers throughout the heating and cooling. The box, already 5 C warmer than ambient because of the V2A BCAM’s self-heating, warmed up by another 5 or 10 C under the bulb over the course of an hour.
We note that the 3-W bulb, being so close to the top of the BCAM, illuminated only a small area of the top, perhaps 2 cm across, and in the center.

150-W at 10 cm, BCAMs Separated by 70 cm

With V2A BCAMs, the 150-W bulb 10-cm from the top of the left BCAM, and the two separated by 70 cm, the image positions in the left BCAM moved by several micrometers. The distance between the two image in the left BCAM changed by one or two micrometers. The following graphs show the changes, with measurements every ten seconds. We heated the BCAM for only 1000 s because it was almost too hot to touch by that time, and we did not want to damage its electronics. The case temperature was at least 60 C. We observed no significant change in the positions of the images in the right BCAM.
The 150-W bulb is 20-cm across, and heats the entire top of the left BCAM. It also heats the right side, although less quickly. The top of the box is at right angles to the incident radiation, but the side is at a small angle.
Analysis

One effect of heat upon the BCAM is the expansion of the aluminum chassis compared to the CCD. Even if the CCD reached the same temperature as the box, its expansion coefficient is close to 7 ppm/K for silicon, as compared to 25 ppm/K for aluminum. With a 1-K rise in temperature, the box expands by 25 ppm, which increases the image separation by 25 ppm. The pixel separation increases by only 7 ppm. If the BCAM warms up by 40 C, we expect the image separation to increase by 1.2 um, or 0.07% from thermal expansion alone. The thermal expansion, however, cannot account for the changes in the image separation when the images are only 140 µm apart.

The following figures are transparencies from a presentation we made at CERN in February 2000 on the subject of atmospheric turbulence and temperature-gradients, and their possible effects upon an optical alignment system. In the talk we derive the following relationship,
\[ s = -L^2 \frac{\partial T}{\partial x} (n - 1) \frac{1}{2T}, \]

where \( s \) is the displacement of a light ray perpendicular to its direction of propagation, \( L \) is the length of a ray, \( x \) is displacement perpendicular to the direction of propagation, \( T \) is air temperature in Kelvin, \( n \) is the refractive index of air at temperature \( T \).

Figure: Bending of Light by Thermal Gradients in Air

To estimate the ray-bending that might occur in our BCAM box as a result of heating it up and cooling it down, let us make a few estimates of the terms in the above equation, as they apply to the BCAM heated by a 150-W bulb, and then allowed to cool. For the heated BCAM we have the top of the box at 60°C and the bottom at 20°C. The BCAM box is about 4 cm high. That gives us a temperature gradient across the box of order \( 10^3 \) K/m. The length of the optical path in the box is 7 cm. Under these circumstances, we expect of order 2.5 µm displacement of the images downwards on the CCD. Downwards on the CCD is
upwards in our images, so we are looking for an upward displacement of the images of order several microns. We see that the top image in the short-range experiment moves up by four microns as the BCAM heats up.
\[ T(x) \]

Sides lose no heat

cross section A

\[ q(x+dx) \]

\[ T(x+dx) \]

\[ q = \text{heat flow} = q(x,t) \]
\[ T = \text{temperature} = T(x,t) \]
\[ t = \text{time} \]

Heat In = Heat Out + Heat Stored

\[ q(x) = q(x+dx) + CA\,dx\,\rho \frac{\partial T}{\partial t} \]

where \( c \) = specific heat capacity \((\text{J K}^{-1} \text{kg}^{-1})\)

\( \rho \) = density

Also, \( q(x) = -x\,A \frac{\partial T}{\partial x} \)

where \( x \) = thermal conductivity \((\text{J s}^{-1} \text{m K}^{-1})\)

So \( q(x+dx) - q(x) = -x\,A \frac{\partial^2 T}{\partial x^2} \, dx \)

Combining and dividing by \( A\,dx \) we have:

\[ \frac{\partial T}{\partial t} = \frac{x}{\rho C} \frac{\partial^2 T}{\partial x^2} \]

Figure: Derivation of the Transient Heat Conduction Equation
More difficult to explain are the changes in the separation of the two images. The separation in the short-range experiment changes by up to two microns. If these changes are due to thermal gradients, the gradient must differ by approximately $10^3 \text{ K/m}$ from the top to bottom of the CCD, which implies a second derivative of temperature with position of order $10^6 \text{ K/m}^2$. But the second derivative with position is related to the first derivative with time by the transient heat conduction equation (see above for derivation),

$$\frac{\partial T}{\partial t} = \frac{\alpha}{\rho C} \frac{\partial^2 T}{\partial x^2},$$

where $T$ is temperature, $t$ is time, $\alpha$ is the thermal conductivity, $C$ is the specific heat capacity, $\rho$ is density and $x$ is position.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$C$ (J kg$^{-1}$ K$^{-1}$)</th>
<th>$\alpha$ (W m$^{-1}$ K$^{-1}$)</th>
<th>$\alpha/\rho C$ (m$^2$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.2</td>
<td>1000</td>
<td>0.026</td>
<td>21 E -6</td>
</tr>
<tr>
<td>Zinc</td>
<td>7000</td>
<td>390</td>
<td>116</td>
<td>42 E -6</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
<td>900</td>
<td>237</td>
<td>98 E -6</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>4200</td>
<td>0.610</td>
<td>0.14 E -6</td>
</tr>
</tbody>
</table>

Table: Thermal Properties of Three Materials

Using the values given for air in the table above, we conclude that if the second derivative of temperature with respect to $x$ is $10^6 \text{ K/m}^2$, the temperature must be changing at 20 K/s. If we assume the temperature inside the box is nowhere greater than 60 C, and nowhere less than 20 C, then such a rate could be sustained in one direction only for two seconds, and yet we observe fluctuations in separation that last for hundreds of seconds. But the thermal gradient in the BCAM enclosure is two-dimensional, so changes in direction of the thermal gradient can generate a first derivative of temperature with position, while the second derivative remains zero. Here is a solution to the transient heat conduction equation, which allows us to estimate the speed with which thermal gradients establish themselves within the enclosure.
One-Dimensional Solution

Apply temperature A here

\[ T(x,t) = \text{temperature} \]

\[ \frac{\partial T}{\partial t} = \frac{x}{\rho c} \frac{\partial^2 T}{\partial x^2} - 0 \]

Boundary Conditions:

\[ t < 0 \Rightarrow T = 0 \quad j (t = 0, x > 0) \Rightarrow T = 0 \quad j (t > 0, x = 0) \Rightarrow T = A \]

Take Laplace Transform of (1) w.r.t. time:

\[ sT(s) - T_0 = \frac{x}{\rho c} \frac{\partial^2 T(s)}{\partial x^2} \]

where \( s \) is dummy operator, \( T_0 \) is \( T(0) \), and \( T(s) = \text{transform of } T \).

For \( x > 0 \) we have \( T_0 = 0 \), so we have

\[ \frac{\partial^2 T(s)}{\partial x^2} = \frac{\rho C_s}{x} T(s) \quad x > 0 \quad \Box \]

For \( x = 0 \) already know that \( T(t) = A h(t) \)
where \( h(t) \) is the Heaviside function, so the solution at \( x = 0 \) is

\[
T(s) = A/s
\]

\[x = 0 \quad (\text{2})\]

The solution to \( (\text{1}) \) and \( (\text{2}) \) is

\[
T(s) = \frac{A}{s} e^{-\sqrt{\frac{s}{\rho c}}}
\]

From which

\[
T(x,t) = A \left(1 - \text{erf} \left( \frac{x}{2\sqrt{\frac{s}{\rho c}}} \right) \right)
\]

where \( \text{erf}(v) = \frac{2}{\sqrt{\pi}} \int_{0}^{v} e^{-u^2} \, du \)

= error function

\[
\text{erf}(v) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n v^{2n+1}}{(2n+1) n!}
\]
The thermal gradient caused by a hypothetical, sudden increase in the temperature of the lid of the BCAM enclosure would reach the BCAM’s optical axis, which is five millimeters beneath the lid, in one second, and arrive at the base of the enclosure in about a minute. When we consider also that thermal gradients travel faster through aluminum than air, and that heat will be lost from the sides of the box by convection, we arrive at a complicated development of thermal gradients within the box over hundreds of seconds, during which thermal gradients can reverse direction. Such a picture seems to us consistent with what
we saw in our 150-W heating experiment at range 70 cm. It is not consistent, however, with our next experiment.

150-W at 20 cm, BCAMs Separated by 10 m

At the request of Joseph Rothberg and Christoph Amelung, we repeated the above experiment, but with the BCAMs separated by 10 m instead of 70 cm. At this greater range, the laser images are separated on the CCD by only 120 µm. We placed the 150-W lamp 20 cm from the BCAM and used two V2B BCAMs instead of the previous V2A BCAMs. We took images every ten seconds. Each measurement requires two images, a foreground and background, so every ten seconds we took four images. The BCAM was awake for approximately 200 ms out of every 10 s. Its sleep-state power consumption is 20 mW, so its average power dissipation during our data taking was approximately 40 mW. After we switched the lamp on, the BCAM warmed up from 22 C to 41 C. The following figure shows the vertical position of each image, with the average value subtracted for presentation.

![Figure: Fluctuation in Vertical Position of Top and Bottom Images.](image)

The following figures show the horizontal fluctuation in position. The first figure gives the variations for the entire experiment, and the second shows in
more detail the fluctuations in the first few minutes after the lamp turns on. In the second figure, the 200-s delay between the horizontal fluctuation of the two images is clearly visible. Note that the top spot is lower down on the actual CCD.

Figure: Fluctuation in Horizontal Position of Top and Bottom Images.
Figure: Detail of Fluctuation in Horizontal Image Position from Long Range Experiment.

Figure: Fluctuation in Separation of Top and Bottom Images
Repeat of 150-W at 20 cm, 10-m Range

We repeated the long-range experiment, to see if the fluctuations would be the same as the first time, and also to store images to disk and examine them individually. The following graph shows the fluctuation in x for the first and second trial, immediately after the lamp turns on.

![Figure: Fluctuation in X-Position of BCAM_LT, shown for first and second trial at 10 m.](image)

We examined the images taken during the second trial. We looked for changes in their shape, and especially asymmetry, but we saw no changes. We re-analyzed the images using intensity thresholds 30, 50, and 60. The analysis with threshold 30 showed a few spikes in position, but other than that, the results and graphs were the same for all values of threshold. We also performed the 10-m experiment with a paper tube around the light path inside the BCAM, but the tube had no observable affect on the movements of the images.

Analysis

In the long-range experiment, with the images separated by only 120 µm, we still see changes in the image separation of order one micron. If thermal gradients are responsible, the second derivative of temperature with position must
now be of order $10^7 \text{ K/m}^2$, and the associated rate of change of temperature with time becomes 200 k/s.

When we insert a paper tube around the optical path, we eliminate with certainty any convection along the path. The tube has no effect upon the movements of the images. The paper tube has conductivity similar to that of air, and it is thin, so we do not expect its insertion into the BCAM to alter the thermal gradients within the box.

150-W at 20 cm, 10-m Range, Sources Moved 5-mm

We moved the right BCAM five millimeters to the left, as seen from the left BCAM. The top and bottom images moved horizontally by only 50 μm. We removed the paper tube in the BCAM, and repeated the long-range experiment. Here is the horizontal position of the top image in its new location after the lamp turned on, compared to its position in the previous location.

![Image: Fluctuation in Horizontal Position of Images, before and after we moved the right BCAM by 5 mm.](image_url)
The following figure shows that the changes in the separation of the two images are still of the same size, but now occur approximately 100 s earlier.

Figure: Fluctuation in Separation of Images, before and after we moved the right BCAM by 5mm.

150-W at 20 cm, 10-m Range, Brass Tube Around Light Path

We put a wooden scaffold inside the BCAM and with it supported a brass tube, of inner diameter 6 mm and outer diameter 10 mm, that enclosed the entire exposed optical path inside the BCAM enclosure, except for 2-mm next to the brass lens holder. We repeated the long-range experiment, leaving the right BCAM in the same position it was in the previous trial.

The following graph shows the horizontal movement of the top image with and without the metal tube. To our astonishment, the horizontal movement of the top spot has the same pattern and size both with and without the metal tube. The only difference between the two graphs is that the fluctuations with the tube inserted are advanced in time by approximately 100 s.
Figure: Affect of Heating on X-Position of Top Spot With and Without Metal Tube. The offset between the two graphs is due to the average value of the position being calculated over 3,000 s for the without-tube experiment, and 76,000 s for the with-tube experiment.

Figure: Image Position and Separation During Heating with Metal Tube Around Light Path.
The long-term horizontal position in the second graph shows that the change in x at turn-off is opposite to that at turn-on. The same is true for y.

**Analysis**

The heat capacity and high conductivity of the brass tube ensure that there will be no thermal gradients in the air along the optical path. The fluctuations cannot be caused by thermal gradients in air inside the BCAM enclosure. The fact that the fluctuations change when we move the starting position of the images by as little as fifty microns appears to point to a systematic error on the CCD that has nothing to do with heat at all.

**Linearity at 10 m**

Without heating, we moved the right BCAM horizontally, perpendicular to the axis of the left BCAM, to test its linearity at 10 m. We fitted a straight line to the x- and y-coordinates of each image. The following plot shows the residuals from these fits.
A periodic non-linearity in the x-coordinates of both spots is clear in the figure. It has period three pixels.

**Linearity with New Focus**

We made a new BCAM with the lens two millimeters closer to the CCD than before. The following two figures show images taken with the original and new lens positions. Note the diffuse ring around the bright center in the 10-m image taken with the original lens position.
We repeated the linearity experiment at three ranges with the new lens position.
Figure: Linearity at 10 m with New Focus. Standard deviation of residuals is less than 0.3 µm for each coordinate and image.

Figure: Linearity at 2 m with New Focus. Standard deviation of residuals is less than 0.3 µm for each coordinate and image.
Figure: Linearity at 1 m with New Focus. Standard deviation of residuals is less than 0.3 µm for each coordinate and image.

To check for cyclic errors with vertical motion on the CCD, we took mounted the newly-focused BCAM on the right-hand stand, and moved it with the micrometer in 200-µm steps, taking images of the stationary left-hand BCAM. We did not expect this experiment to yield good linearity, because when the BCAM itself moves on a stage, it can rotate as well as translate. Nevertheless, the separation of the two images should still be constant.
We conclude that the diffuse ring around the images obtained from the original BCAM are responsible for the cyclic non-linearity we observed at 10 m. We still see some sign of a cyclic error with the new BCAM at 10 m, but its amplitude is not more than 0.3 µm.
While we were moving the right-hand BCAM, we noticed that it was consistently generating bad measurements in certain positions towards the end of the scan. We took the following image with a 300-ms exposure.

![Reflection in a BCAM Image, 300-ms exposure, with strong intensification.](image)

Figure: Reflection in a BCAM Image, 300-ms exposure, with strong intensification. We can see one strong reflection and two smaller ones, in addition to the main image.

Given that many BCAMs will be looking up and down shiny alignment bars, we may find that such reflections occur in many of our H8 and ATLAS image. But we can always identify the reflection, if only because it is so much less intense than the correct image. When we know where the reflection is, we can place our analysis bounds so as to exclude it, and thus obtain an accurate measurement.

150-W at 20 cm, 10-m Range, New Focus

We repeated our heating experiment, without a metal tube, but with the newly-focused spot. After several aborted runs, we discovered that we had to reduce the maximum and minimum intensities of the light spots, because when the CCD in this particular BCAM became hot, the saturation level of the pixels dropped from 180 counts to 90 counts. Here is an intensified dark current image
taken from the BCAM when it was hot. We put a piece of paper over the lens to make sure no light contaminated our picture of the dark current. The exposure time was 10 ms. The maximum intensity is 55 counts at the top of the image, and 37 counts at the bottom. A zero-second exposure had intensity 27 at the top and 17 at the bottom. The fact that the dark current does not vary linearly from top to bottom suggests that the CCD temperature is not uniform. But we do not obtain similar images from other CCDs.

Figure: Dark Current from the Hot CCD, an intensified image with fluctuations in row-intensity plotted in yellow, and fluctuations in column-intensity plotted in red.

We note that we have not observed the same behavior in any other CCD, but we are concerned that one CCD should differ from the others. Nevertheless, with our new DAQ settings, we performed the experiment successfully. The following graph shows the fluctuation in image separation after we turned the lamp on. The standard deviation of the separation for the first thousand seconds is 0.4 µm, down from 0.8 µm with the original BCAM.
The following figure shows the movement of the images.

We were surprised to see such small changes in image position, so we made another BCAM to see if it would behave the same way.
We made another BCAM, put the lens in the new position, and captured an image. The image showed a dim, diffuse cloud around it. We checked the new BCAM’s linearity and observed the 3-pixel period cyclic error. We knocked the lens out with a screwdriver, and put a new one in. There was no cloud around the image, and we went ahead and repeated the heating experiment. We obtained the following fluctuations in position. The separation of the images remained constant to 0.4 µm.

![Fluctuations in Image Position for the Second Newly Focused BCAM](image.png)

Given that we now see four-micron changes in the horizontal position, we assume that the exact change in image position caused by heating depends upon the BCAM.

**Conclusion**

The changes in spot separation that occurred when we and our colleagues heated our BCAMs were caused not by the heating directly, but by defocus of the coherent-light image. With the image focusing several millimeters in front of the CCD, we see a cloud around the central bright spot. When this cloud is present,
we observe cyclic errors in our measurement of the spot position. These cycles have period three pixels and amplitude up to 1.5 µm. We do not understand these errors, although we suspect an interaction between interference fringes and the pixel structure. We will investigate as soon as we can.

We eliminated the cyclic errors by moving the lens so that the image plane was no more than a millimeter in front of the CCD at all ranges, although it was several millimeters behind the CCD at close ranges. Focusing behind the CCD does not appear to introduce cyclic errors.

With the cyclic errors gone, we repeated our heating experiments, and observed the positions of both images moving together, as we would expect from deformation of the BCAM enclosure. We eliminated thermal gradients in the air as the cause of these motions when we earlier placed a metal tube around the optical path within the BCAM.

With correct focus, the BCAM performs well at ranges 1 m to 10 m. We will test it at shorter ranges when we have time. But correct focus cannot be obtained with the lenses in our possession simply by putting each lens in the same place with respect to the enclosure. The lenses, it would appear, have different focal lengths. When making the remainder of the BCAMs for the H8 test stand, we plan to move each lens gradually into the enclosure while capturing live images of a source ten meters away. When the cloud around the image disappears, we will glue the lens, but we will not move the lens any closer than is necessary to remove the cloud. By this procedure, we will guarantee performance at long ranges, while providing the best performance possible at short ranges.